Title: Equilibrium Between Rotation and Gyration in the Precession of Pencil Caps

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This study examines the dynamics of a spinning pencil cap on a smooth surface—a motion commonly observed during childhood play. As the cap spins about its axis, it simultaneously traces a circular path and gradually tilts down. This behavior represents a form of precession arising from the interaction between axial rotation and gyration. This study aims to formulate and evaluate a mathematical model that accurately represents this stable and continuous precession, thereby elucidating this complex physical phenomenon. A mathematical model that characterizes the interplay between rotation and gyration is constructed. This model achieves a balance between the rotational velocity (Vs) and contact point velocity (Vp). When Vp < Vs, the cap tilts up, and the rotational speed decreases; conversely, when Vp > Vs, it topples and accelerates. This continuous interplay induces oscillations with the inclination angle $\theta$ between the spinning cap and the ground. At equilibrium, $V_p$ and $V_s$ become equal. If one velocity temporarily dominates, $V_s$ adjusts to restore balance, leading to a self-regulating dynamical state. Experimental validation is performed utilizing a 50-mm long acrylic pipe weighted with 2.4 g of Pb on a flat tabletop. Side-view video recordings are analyzed to determine the angular frequency of axial rotation and inclination angle $\theta$ between the spinning pipe and the tabletop. The measurements show that $\theta$ exhibits oscillatory behavior during motion, supporting the existence of a dynamic balance between rotational and gyrational velocities and thereby validating. This phenomenon has the potential to serve as a familiar teaching aid for explaining precession.